Analytical methods in interplay with physics

A workshop to explore Asymptotics, Randomness and Noncommutativity

27-29 November 2024, Potsdam

Organizers:

Onirban Islam (Potsdam) Gihyun Lee (Potsdam) Max Lein (Potsdam)

Sylvie Paycha (Potsdam) Nikolas Tapia (Berlin) Fabrizio Zanello (Potsdam)

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Early morning in the Sans-Souci park nearby
Picture by S. Paycha

ANALYTICAL METHODS IN INTERPLAY WITH PHYSICS

Speakers

Program

Day 1 (November 27th): Analysis & randomness	09:00 - 09:50	Lorenzo Zambotti
	10:00 - 10:40	Alberto Bonicelli
	10:50 - 11:10	COFFEE BREAK
	11:10 - 11:50	Carlo Bellingeri
	12:00 - 12:40	Jean-David Jacques
	12:40 - 14:00	LUNCH BREAK
	14:00 - 14:50	Ilya Chevyrev
	15:00 - 15:40	Harprit Singh
	15:50 - 16:10	COFFEE BREAK
	16:10 - 16:50	Sarah-Jean Meyer
	17:00 - 17:40	Benjamin Gess
	18:00 -	WORKSHOP DINNER *Augustiner im Bürgerbahnhof*

Day 2 (November 28th): Asymptotic & spectral analysis	09:00 - 09:50	Alexander Strohmaier
	10:00 - 10:40	Alessandro Contini
	10:50 - 11:10	COFFEE BREAK
	11:10 - 11:50	Arne Hofmann
	12:00 - 12:40	Ram Band
	12:40 - 12:50	CONFERENCE PICTURE
	12:50 - 14:00	LUNCH BREAK
	14:00 - 14:40	Clara Aldana
	14:50 - 15:30	Michele Fantechi
	15:40 - 16:00	COFFEE BREAK
	16:00 - 16:10	Yannik Vargas
	16:12 - 16:22	Binh Tran
	16:24-16:34	Camilo Angulo
	16:36-16:46	Mir Mehedi Faruk
	16:48-16:58	Sajad Aghapour
	17:00-17:10	Rebecca Roero
	17:12-17:22	Alejandro Peñuela Diaz

Day 3 (November 29th): Noncommutative geometry & topological insulators	09:00 - 09:50	Nils Waterstraat
	10:00 - 10:40	Hermann Schulz-Baldes
	10:50 - 11:10	COFFEE BREAK
	11:10 - 11:50	Nora Doll
	12:00 - 12:40	Eske Ewert
	12:40 - 14:00	LUNCH BREAK
	14:00 - 14:50	Michał Eckstein
	15:00 - 15:40	Arkadiusz Bochniak
	15:50 - 16:30	Tomasz Miller
	16:40 - 16:50	CLOSING REMARKS
	17:30 -	GATHERING AT PUB *Cuhibar*

Abstracts

Clara Aldana

Independent Researcher

TITLE: **Quasi-isospectral Schrödinger operators.**

In this talk I will discuss quasi-isospectral Schrödinger operators (Sturm-Liouville) on finite intervals and on closed Riemannian manifolds. I will introduce quasi-isospectrality as a generalization of isospectrality. I will briefly mention some historical facts about the problem and the problems one finds to construct quasi-isospectral potentials on the interval. This talk involves asymptotic expansions of heat traces and heat invariants. The work presented here is based on ongoing joint work with Camilo Perez.

Ram Band

Technion - Israel Institute of Technology, Haifa, and Universität Potsdam

TITLE: **The Dry Ten Martini Problem for Sturmian Schrödinger operators.**

"Are all gaps there", asked Mark Kac in 1981 during a talk at the AMS annual meeting, and offered ten Martinis for the answer. This led Barry Simon to coin the names the Ten Martini Problem (TMP) and the Dry Ten Martini Problem for two related problems concerning the Almost-Mathieu operator. The TMP is about showing that the spectrum of the Almost-Mathieu operator is a Cantor set. The Dry TMP is about the values that the integrated density of states (IDS) attains at the spectral gaps. The gap labelling theorem predicts the possible set of values which the IDS may attain at the spectral gaps. The Dry TMP is whether or not all these values are attained, or equivalently, "are all gaps there?". We present an affirmative solution to the Dry Ten Martini Problem for Sturmian Hamiltonians. Concretely, it is proved that all spectral gaps are there for Schrödinger operators with Sturmian potentials and non-vanishing coupling constant. The talk is based on a joint work with Siegfried Beckus and Raphael Loewy.

Carlo Bellingeri

Institut Élie Cartan de Lorraine, Nancy

TITLE: **Symmetries for the gKPZ equation via multi-indices.**

In this talk, I will present how to use the recent theory multi-indices by Otto and authors to compute the dimension of the two spaces associated with the symmetries of the one-dimensional generalised KPZ equation (gKPZ): the chain rule and the Itô Isometry. Our proof is quite elementary and shows that multi-indices provide in this case a simplification in comparison to the results obtained via decorated trees. Joint work with Y. Bruned (Université de Lorraine).

Arkadiusz Bochniak

Max-Planck-Institut für Quantenoptik, Garching

TITLE: **Spectral functionals for geometries with torsion.**

Spectral methods of noncommutative geometry allow for studying characteristics of quantum analogs of Riemannian geometries, such as their curvature. Noncommutative versions of geometries equipped with the Levi-Civita connection are nowadays well-studied; however, the Cartan geometry and the Weitzenböck connection are still not satisfactorily explored. We study spectral functionals (spectral action, Einstein, and torsion functional) for geometries with nonvanishing torsion, both for spin geometries and ones based on the Hodge-Dirac operator. Based on a work in progress with L. Dabrowski, A. Sitarz and P. Zalecki.

Alberto Bonicelli

Sorbonne Université, Paris

TITLE: **Renormalization as an extension problem.**

Taking the moves from the review of Prof. Zambotti on the renormalization problem in quantum field theory, I will comment on how, from a perturbative point of view, it boils down to extending distributions over singular support. Starting from basic theorems on the extension of quasi-homogeneous distributions at a point, I will go through two examples which clearly show where the renormalization freedom lies. If time permits, I will also address a perturbative study of the moments of the solution to a singular SPDE, showing how renormalization again amounts to the extension of suitable integral kernels and how this links with other non-perturbative procedures.

Ilya Chevyrev

University of Edinburgh

TITLE: **Well-posedness of subcritical non-linear heat equations.**

In this talk, I will present a recent work showing local well-posedness of heat equations with polynomial non-linearities and subcritical Gaussian initial conditions. This result is motivated by the problem of constructing state spaces for geometric QFTs and stochastic PDEs close to criticality. The method of proof is divided into two parts, one deterministic and one probabilistic. In the deterministic part, we construct a non-linear metric space of distributions which acts as a good space of initial conditions for the PDE. In the probabilistic part, we show that suitable Gaussian fields take values in this metric space. We achieve this with a combinatorial argument to inductively bound Feynman diagrams arising from the Wild expansion of the solution. Based on joint work with Hora Mirsajjadi.

Alessandro Contini

Leibniz Universität Hannover

TITLE: **Eigenvalue asymptotics for the Dirichlet extension.**

Eigenvalue asymptotics for the Laplacian have a long history and relate to different topics across maths. The most well-known setup is for the Laplace-Beltrami operator on a closed manifold, with the formula dating back to H. Weyl (indeed, this is how it is usually known). In this talk I will try to sketch an approach to the asymptotics for the Dirichlet Laplacian based on the wave trace, following works of many authors (Ivrii, Hörmander and others).

Nora Doll

Martin-Luther-Universität Halle-Wittenberg

TITLE: **Skew localizer for real index pairings.**

In this talk index pairings of a projection and a unitary where both, the projection and the unitary fulfill real symmetry relations are considered. For a given combination of symmetries the integer-valued index of the pairing vanishes, but there may be a $\mathbb{Z}_2$-index given by the dimension of its kernel, modulo 2. The aim is then to construct a finite-dimensional real skew-adjoint matrix called the skew localizer for these pairings and to show that the $\mathbb{Z}_2$-index can be computed as the sign of the Pfaffian of the skew localizer. The main tool to prove the connection of the $\mathbb{Z}_2$-index to the sign of the Pfaffian of the skew localizer is the orientation flow of paths of bounded real skew-adjoint Fredholm operators. This orientation flow might be of independent interest and is introduced in the second part of this talk.

Michał Eckstein

Uniwersytet Jagielloński w Krakowie

TITLE: **From physics to noncommutative geometry and back.**

Modern physical theories draw handfuls of structures from differential geometry. This sounds fairly intuitive when one thinks of general relativity (aka spacetime geometry) or classical mechanics, but how about quantum theory? While the phase space of a classical system (spanned by the available positions and momenta of particles) is a space indeed, for a quantum system this is no longer the case. The positions and momenta of quantum particles naturally determine a noncommutative algebra rather than a space.

Noncommutative geometry arose from the quest to understand the geometry of the quantum. Pondered already by Werner Heisenberg (one of the founding fathers of quantum mechanics) in 1930s, it acquired a more concrete shape only at the end of the previous century. In essence, noncommutative geometry is concerned with seeking analogues of differential-geometric structures in the realm of complex noncommutative algebras. Within the lecture we shall focus on noncommutative geometry as put forward and developed by Alain Connes.

We shall start the lecture with reviewing the major physical and mathematical motivations behind noncommutative geometry. Then, we will get familiar with the basic structure (a spectral triple) through definitions and examples. Next, we will encounter the spectral action principle, its implementation and tools for computations. The latter would bring us back to physics. We shall discuss some modern applications of noncommutative geometry in particles physics, as well as the ambitious challenge to build a consistent theory of quantum spacetimes.

Eske Ewert

Leibniz Universität Hannover

TITLE: **Shubin calculus on graded Lie groups.**

To obtain a better understanding of hypoelliptic differential operators which are not elliptic in the classical sense, one can consider a calculus where vector fields can have order higher than 1. For example, on the Heisenberg group one can give order 1 to X, Y and order 2 to $Z = [X, Y]$ to obtain a calculus in which the Sublaplacian is 'elliptic'. In this talk, I will discuss a variant of Shubin's calculus on graded Lie groups using this notion of order. We show that elliptic operators in this pseudodifferential calculus are Fredholm on these noncompact groups. Similar to the approach by van Erp and Yuncken we use an appropriate groupoid to define the calculus. In particular, we show that several generalizations of the harmonic oscillator to the Heisenberg group are elliptic in our calculus. This talk is based on joint work with Philipp Schmitt (arXiv:2407.14347).

Michele Fantechi

Politecnico di Milano

TITLE: **Quasi-classical limit of the Caldeira-Leggett model.**

The Caldeira-Leggett model describes a quantum particle coupled to a reservoir of infinitely many oscillators that act as a heat bath. This fundamental toy model was proposed to analyze dissipation and decoherence in open quantum systems. We derive an effective dynamical map for the quantum particle by performing a partial classical limit of the bosonic field by infinite-dimensional semiclassical analysis techniques, which correspond to a regime of a large number of excitations in the field. According to the effective dynamics, the field evolves freely, and there is no back-reaction from the particle. In contrast, the reduced density matrix of the particle's evolution is an average of unitary dynamics generated by an effective Hamiltonian that depends on the field state.

Benjamin Gess

Technische Universität Berlin

TITLE: **From non-equilibrium statistical mechanics, to PDEs with irregular coefficients, to gradient flow structures.**

In this talk, we examine the connections between non-equilibrium statistical mechanics, represented by macroscopic fluctuation theory, large deviation principles, PDEs with irregular coefficients, and gradient flow structures. As a model example, we analyse the large deviation behaviour of the rescaled zero-range process in relation to its hydrodynamic limit, governed by the porous medium equation. This involves an investigation of the skeleton equation, an energy-critical, degenerate parabolic-hyperbolic partial differential equation with irregular drift. We subsequently explore the link between large deviation principles and the formal gradient flow interpretation of the porous medium equation, deriving an entropy dissipation equality from the principles of large deviations and reversibility.

Arne Hofmann

Leibniz Universität Hannover

TITLE: **Relative trace formulas in spectral geometry and physics.**

Wave trace invariants are spectral invariants that can be used to study both compact and noncompact manifolds. In the case of scattering by a finite number of compact obstacles, it is useful to consider a variant, the relative wave trace. Its leading singularities are related to the asymptotic decay of the function Xi, which has been studied by (Fang-Strohmaier, 2021) and (Hanisch-Strohmaier-Waters, 2022). The function Xi also appears as the energy term which causes the Casimir effect in physics. In this talk, I will introduce the function Xi, its connections to geometry and physics, and remark on recent progress concerning non-Dirichlet boundary conditions, especially transmission boundary conditions.

Jean-David Jacques

Universität Potsdam

TITLE: **Post-Lie algebras in regularity structures.**

In this talk I will give a short introduction to post-Lie algebras and I will explain how they permit to construct the structure group for stochastic PDEs in the framework of the theory of regularity structures.

Sarah-Jean Meyer

University of Oxford

TITLE: **A forward-backward stochastic differential equation and a martingale problem for sine-Gordon.**

I will present a new characterization of the sine-Gordon (SG) quantum field theory (QFT) based on a martingale problem. The argument builds on our previous results on the SG QFT using forward-backward SDEs, so I will first take some time to explain the general idea of the FBSDE approach. From there, I will formulate a martingale problem for the sine-Gordon QFT. Under certain conditions, the martingale problem gives a unique characterization of the measure. This is based on joint work in progress with Massimiliano Gubinelli.

Tomasz Miller

Uniwersytet Jagielloński w Krakowie

TITLE: **Causal evolution of probability measures and continuity equation.**

One of the most important concepts of relativistic physics is that of a world line, i.e., the space-time trajectory of a point particle. In Lorentzian geometry, it is modelled by the so-called causal curve, and the questions concerning which spacetime points can be connected by means of causal curves lead to a vast area of study known as causality theory. In the talk, I will present how the basic notions of causality theory can be naturally extended to probability measures on spacetimes (what is motivated by both classical and quantum physics, but also by mathematics itself). Furthermore, I will discuss the notion of a causal evolution of probability measures, its deep connection with the continuity equation and some surprisingly nice topological properties of the space of causal curves.

Hermann Schulz-Baldes

Friedrich-Alexander-Universität Erlangen

TITLE: **Topological invariants in solid state systems.**

The talk will give an overview on topological insulators, with a particular focus on interrelation between topological invariants, their index theory and the spectral localizer.

Harprit Singh

University of Edinburgh

TITLE: **Singular SPDEs on homogeneous Lie groups.**

The theory of regularity structures provides a general and robust framework to understand the well posedness of subcritical parabolic SPDEs. I shall explain that by interpreting the base space as a (non-commutative) homogeneous Lie Group one can extend the scope of this theory to a large class of hypo-elliptic/ultra-parabolic SPDEs, in particular covering the case of equations where the differential operator is the kinetic equation $\partial_t - \triangle_v - v \cdot \nabla$ or a heat type operator $\partial_t - \sum_i X_i^2$ on a Carnot group. By combining classical results about hypo-elliptic operators with the theory of regularity structures, this is achieved in high generality without "reinventing the wheel".

Alexander Strohmaier

Leibniz Universität Hannover

TITLE: **Local trace formulae and quantum ergodicity.**

The talk aims at non-specialists with interest in spectral theory. I will review various trace formulae in spectral geometry and how they relate to quantum ergodicity and Weyl's law. I will briefly discuss a setting of spectral geometry applicable in general relativity but will on the whole be focused on classical results such as the Duistermaat-Guillemin trace formula and the Shnirelman-Colin-de-Verdiere-Zelditch quantum ergodicity result.

Nils Waterstraat

Martin-Luther-Universität Halle-Wittenberg

TITLE: **The G-equivariant spectral flow and bifurcation of functionals with symmetries.**

The aim of the talk is to give an idea how the spectral flow can be used to find bifurcation points from a trivial branch for critical points of one-parameter families of functionals. In particular, we outline a new research line, where functionals with symmetries are considered and the classical spectral flow needs to be replaced by an equivariant version of it.

Lorenzo Zambotti

Sorbonne Université, Paris

TITLE: **Renormalisation from quantum field theory to stochastic partial differential equations.**

In the last decade there has been a lot of work on singular Stochastic Partial Differential Equations (SPDEs) requiring a renormalisation, namely a procedure based on a regularization of the noise and a deformation of the equation by subtracting suitable counterterms. These counterterms have a precise structure and are very well described algebraically; they contain the famous diverging constants (infinities) which make renormalisation a mysterious and fascinating topic and appear famously in Quantum Field Theory (QFT), where they are related to ultra-violet divergences. Theoretical physicists introduced SPDEs in QFT in the 80s in order to give a new constructive approach to Euclidean quantum fields. In this talk I will try to compare these two theories, showing that there are similarities but also important differences, such that it is still broadly impossible to fully translate the results of one theory in the context of the other.

GONG TALKS

Sajad Aghapour

Albert Einstein Institute, Golm

TITLE: **Complex transformations in GR and field theories.**

Complex transformations are novel methods that can relate solutions of field theories with different parameters. On good days, they allow us to perform shortcut calculations in complex scattering problems. In my talk, I discuss some of these complex transformations and their applications in GR and field theories.

Camilo Angulo

Jilin University, Changchun, and Georg-August-Universität Göttingen

TITLE: **Gray stability for contact groupoids.**

Contact groupoids are finite-dimensional models "integrating" local Lie brackets on spaces of sections of a line bundle. These brackets are known as Jacobi brackets and they encode time-dependent mechanics in the way Poisson brackets do mechanics. In this talk, we explain how under a certain compactness hypothesis, one can adapt the Gray-Moser argument to these contact structures and point out some applications.

Mir Mehedi Faruk

McGill University, Montréal

TITLE: **Black holes in de Sitter: observers to causal structure.**

We analyze null- and spacelike radial geodesics in Schwarzschild-de Sitter spacetime connecting two conjugate static sphere observers, i.e. free-falling observers at a fixed radius in between the two horizons. We explicitly determine the changes in the causal structure (Penrose diagrams) with respect to these natural observers as a result of the inward bending of the black hole singularity, as well as the outward bending of asymptotic infinity. Notably, the inward and outward bending changes as a function of the black hole mass, first increasing towards a maximum and then decreasing to vanish in the extreme Nariai limit.

Alejandro Peñuela Diaz

Universität Potsdam, Germany

TITLE: **Local foliations by critical surfaces of the Hawking energy and small sphere limit.**

The Hawking energy is one of the most famous local energies in general relativity, by using a LyapunovSchmidt reduction procedure we construct unique local foliations of critical surfaces of the Hawking energy on initial data sets. Any quasilocal energy should satisfy the so-called small sphere limit, therefore we also discuss the relation between these surfaces and the small sphere limit. In particular, we discuss some discrepancies on the small sphere limit, so when approaching a point with these foliations and when approaching as in the small sphere limit.

Rebecca Roero

Humboldt Universität Berlin and Universität Potsdam

TITLE: **Eta invariants of the Berger spheres.**

The aim of this poster is to present an innovative way to compute the eta invariants for the Dirac operator of the Berger spheres. We use the Atyiah-Patodi-Singer theorem for the index of the classical Dirac operator on a manifold with boundary to compute the eta invariant of the boundary. We apply such formula to the case where the manifold with boundary is a closed ball in $\mathbb{CP}^n$, finding the eta invariants for the Berger spheres. We also explain what is the motivation behind the study of the Berger spheres case and some of the possible extensions of this project that we are currently exploring.

Binh Tran

Leibniz Universität Hannover

TITLE: **K-theoretic classification of topological semimetals and interacting topological phases.**

The classification of non-interacting topological insulator phases via K-theory is well-established, but extending this framework beyond topological insulators remains an open challenge. In this talk, I will review approaches that apply K-theory to classify topological semimetals and interacting topological phases.

Yannik Vargas

CUNEF Universidad, Madrid

TITLE: **Connes-Kreimer Hopf algebra and species with grafting operators.**

Inspired by the grafting operator of forests, we define grafting operators for combinatorial species. We show that these operators allow for the construction of up and down operators for species, which are combinatorial analogues of graded vector spaces with creation and annihilation operators. As an application, we reformulate the universal property of rooted forests within the framework of species with a grafting operator. Finally, we show how these operators allow for the construction of balanced pairs of up and down operators. This is ongoing joint work with Pierre Clavier and Sylvie Paycha.